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Study & Analysis of A Cantilever Beam with Non-linear Parameters for Harmonic Response

#1Supriya D. Sankapal, #2Arun V. Bhosale

[1]sankpal.supriya88@gmail.com

[2]arunbhosale@rediffmail.com

[1]Research Scholar, Department of Mechanical Engineering, ACEM, Pune, India.

[2]Assistant Professor, Sinhgad Academy of Engineering, Pune, India

ABSTRACT

A beam is an elongated member, usually slender, intended to resist lateral loads by bending. Structures such as antennas, helicopter rotor blades, aircraft wings, towers and high rise buildings are examples of beams. For beams undergoing small displacements, linear beam theory can be used to calculate the natural frequencies, mode shapes and the response for a given excitation. However, when the displacements are large, linear beam theory fails to accurately describe the dynamic characteristics of the system.

This investigation focuses in the study of the vibration analysis of without crack and cracked cantilever beam subjected to free and harmonic excitation at the base. The objective of the study is to identify the effect of non-linearity namely Material, Geometric and Damping on the natural frequency and mode shapes of cracked cantilever beam by theoretical, numerical and experimental methods.

Keywords— **Cracked cantilever beam, Natural Frequency, Mode Shapes, Linear beam theory, Effect of non-linearity, Free Vibration.**

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I. Introduction

A beam is an elongated member, usually slender, intended to resist lateral loads by bending [1]. Beam elements bend under the loading. Under static loading conditions any system behaves in linear relation. The beam structures are generally designed for static linear load conditions. But beams used in real life conditions are typically subjected to dynamic loads.

For beams undergoing small displacements, linear beam theory can be used to calculate the natural frequencies, mode shapes, and the response for a given excitation. However, when the displacements are large, linear beam theory fails to accurately describe the dynamic characteristics of the system. Highly flexible beams, typically found in aerospace applications, may experience large displacements. These large displacements cause geometric and other nonlinearities to be significant. The nonlinearities couple the (linearly uncoupled) modes of vibration and can lead to modal interactions where energy is transferred between modes [2].

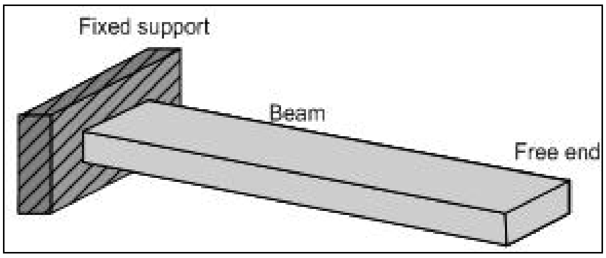

Fig 1 Cantilever Beam

Vibrations are time dependent displacements of a particle or a system of particles w.r.t an equilibrium position. If these displacements are repetitive and their repetitions are executed at equal interval of time w.r.t equilibrium position the resulting motion is said to be periodic.

Classification of Vibration [11]:

Free and forced vibration: If a system, after an internal disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration.

If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as resonance occurs, and the system undergoes dangerously large oscillations. Failures of such structures as buildings, bridges, turbines and airplane have been associated with the occurrence of resonance.

Undamped and damped vibration: If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as undamped vibration. If any energy lost in this way, however, it is called damped vibration.

Linear and nonlinear vibration: If all the basic components of vibratory system—the spring, the mass and the damper—behave linearly, the resulting vibration is known as linear vibration. If, however, any if the basic components behave non-linearly, the vibration is called non linear vibration [11].

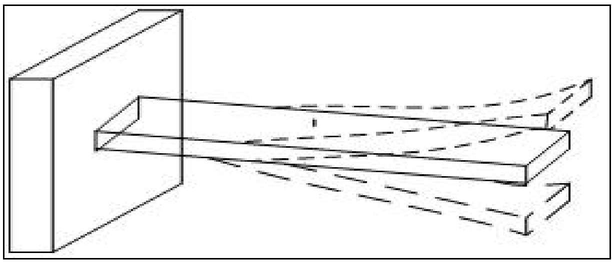

Fig 2 The Free vibration of cantilever beam

II. LITERATURE REVIEW

A wealth of literature exists in the area of vibrations of beams. Some important literatures are given below:

The application of experimental modal testing to various beams based on the impact hammer excitation is attempted to assess the natural frequency, damping constant and associated mode shapes. The modal testing has proven to be an effective and non-destructive test method for estimation of dynamic characteristics of beams [3].

Concept of geometric nonlinearity in mechanics of materials is mentioned. The ANSYS program is used to numerically evaluate the system and calculate Young's modulus of the beam material [4].

By using the set governing flap wise bending motion which was uncoupled with other two sets, flap wise bending .The equations of motion of a rotating cantilever beam are derived based on a new dynamic modelling method. A modal formulation method is also introduced to calculate the tuned angular speed of a rotating beam at which resonance occurs [5].

By considering a nonlinear model for the fatigue crack, the governing equation of motion of the cracked beam is solved using perturbation method. The solution of the governing equation reveals the super harmonics of the fundamental frequency due to the nonlinear effects in the dynamic response of the cracked beam [6].

The governing equations for linear vibration of a rotating Euler beam are derived by the d'Alembert principle, the virtual work principle and the consistent linearization of the fully geometrically nonlinear beam theory in a rotating coordinate system. Governing equations for linear vibration of a rotating inclined Euler beam are derived [7].

Natural frequencies of nonlinear coupled planar vibration are investigated for axially moving beams in the supercritical transport speed ranges. The finite difference scheme is developed to calculate the non-trivial static equilibrium. The equations are cast in the standard form of continuous gyroscopic systems via introducing a coordinate transform for non-trivial equilibrium configuration. The effects of material parameters and vibration amplitude on the natural frequencies are investigated through parametric studies [8].

Analytical solutions of natural frequencies and critical buckling load are obtained for cracked functionally graded materials (FGMs) with clamped-free, hinged-hinged, and clamped-clamped end supports [9].

The perspective of wave method is adopted for the analysis. The method considers the nature of the propagation and reflection of the waves along the beam. Consequently, the propagation, transmission and reflection matrices for various discontinuities located on the beam are derived. Such discontinuities may include crack, boundaries or change in section. By combining these matrices a global frequency matrix is formed. In order to investigate the effect of the beam's structural synthesis, different natural frequencies are obtained and studied [10].

By the literature review it is seen that, compare with previous old systems of vibration analysis of cantilever beam this method identifies the nonlinearities and effects on load deflection characteristics of cantilever.

III. PROBLEM DEFINITION

To perform the Vibration Analysis of Cracked Cantilever Beam of square cross section subjected to Free and Harmonic Excitations. In order to find out effect of various inherent nonlinearities on vibrational behaviours of the beams. Development of experimental setup for validation of the Theoretical and Numerical Analysis will be done.

Objective of the study is to Compare of Theoretical, Numerical and Experimental Vibrational analysis of Cracked Cantilever Beam of square cross section. To find out effect of various inherent nonlinearities on vibrational behaviors of the beams.

Scope is limited to do theoretical study and do Numerical analysis of a cantilever beam without Crack subjected to free Vibrations and Harmonic Excitation with linear and nonlinear parameters. And then Development of experimental setup to carryout vibrational analysis of cracked cantilever beam subjected to free vibrations.

IV. METHODOLOGY

A literature study performed in order to understand the concept of non-linearity in beams. Experimental Analysis of load deflection characteristics of cantilever beam without and with crack will be studied and effect of nonlinearities will be evaluated in terms of their effect on young's modulus. Numerical vibrational analysis of cantilever beam with crack and without crack for free and harmonic excitation will be carried out considering nonlinear system by using young's modulus evaluated by experimental analysis. Experimental setup and methodology will be developed for experimental validation of results obtained by theoretical and numerical method with the help of FFT Analyzer for Free vibration of cracked cantilever beam with nonlinear parameters.

After all the above said procedures just we need to Compare the results of Theoretical, Numerical and Experimental analysis to evaluate the effect of nonlinearities present in the system on Vibrational behaviour of cracked cantilever beam.

Parameter selection

Various types of cantilever beam sections will be studied and one section will be opted for the research work.

Various types of Excitation systems will be studied and one section will be opted for the research work.

- Cantilever Beam without crack
- Cantilever beam with crack
- Cross section of cantilever beam
- Length of cantilever beam

V. PROJECT DESIGN

Feasibility study is done to check economic, technical and operational feasibility.

Theoretical calculations [11]:

Modal analysis is a worldwide used methodology that allows fast and reliable identification of system dynamics in complex structures.

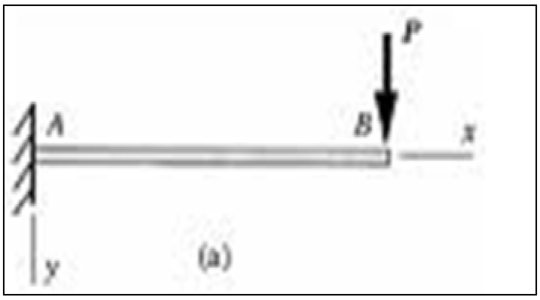

Fig 4 (a) Cantilever beam subjecting a point load P

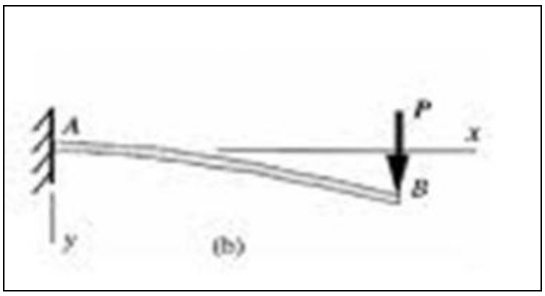

Fig 4 (b) Cantilever beam with small deflection

$$M = EI\frac{\partial^2 y}{\partial x^2} \qquad (1)$$

Where E is Young's Modulus and I is moment of inertia of the beam. The Equation (1) is based on the assumptions that the material is homogeneous, isotropic, obeys Hooke's law and the beam is straight and of uniform cross section. This equation is valid only for small deflection and for beams that are long compared to cross sectional dimensions since the effects of shear deflection are neglected.

Consider a cantilever beam; it is subjected to a point load P therefore the beam will deflect into a curve. When the force, P, is removed from a displaced beam, it will return to its original shape. However inertia of the beam will make the beam to vibrate about its initial location. The equation of beam is

$$\frac{EI}{\rho A}\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = 0 \qquad (2)$$

Where, ρ is the mass density and A is cross sectional area of beam.

$$C^2\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} \qquad (3)$$

I.

Where,

$$C = \sqrt{\frac{EI}{\rho A}}$$

The solution of Eq. (2) is to separate the variables one depends on position and another on time.

$$y = W(x)\,T(t) \qquad (4)$$

By substituting Eq. (4) to Eq. (3), and simplifying, the Equation is:

$$\frac{C^2}{w(x)}\frac{\partial^4 y}{\partial x^4} = -\frac{1}{T(t)}\frac{\partial^2 T(t)}{\partial t^2} \qquad (5)$$

The Eq. (5) can be written as two separate differential equation.

$$\frac{\partial^4 w}{\partial x^4} - \beta^2 W(x) = 0 \qquad (5a)$$

$$\frac{\partial^2 T}{\partial t^2} + \omega^2 T(t) = 0 \qquad (5b)$$

Where,

$$\beta^4 = \frac{\omega^2}{c^2} = \frac{\rho A \omega^2}{EI} \qquad (6)$$

To find out the solution of Eq. (5a), Consider the equation

$$W(x) = C_1 \cos h\beta x + C_2 \sin h\beta x + C_3 \cos h\beta x\, h + C_4 \sin h\beta x$$

In order to solve Eq. (7) the following boundary conditions for cantilever beam are needed:

1. At x=0 → W=0
2. At x=0 → W^{I}=0
3. At x=L→ W^{II}=0
4. At x=L→ W^{III}=0

By substituting boundary conditions into W^{I}, W^{II}, W^{III}.
We obtain the following values of C1, C2, C3, and C4:

$$[\cos h\beta l + \cos \beta l]C_3 + [\sin h\beta l + \sin \beta l]C_4 = 0$$
$$[\sin h\beta l - \sin \beta l]C_3 + [\cos h\beta l + \cos \beta l]C_4 = 0 \qquad (8)$$

We can write Eq. (8) in matrix form as

$$\begin{bmatrix} \cos h\beta l\ + & \cos\beta l + & \sin h\beta l + \sin\beta l \\ \sin h\beta l\ - & \sin\beta l - & \cos h\beta l + \cos\beta l \end{bmatrix} \begin{bmatrix} C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (9)$$

For solving matrix of Eq. (9), we get determinant

$$(\cos h\beta l + \cos\beta l)^2 - (\sin h\beta l + \sin\beta l)(\sin h\beta l - \sin\beta l) = 0$$

$$\cos h^2\,\beta l + 2\cos h\beta l + \cos^2\beta l - \sin h^2\,\beta l + \sin^2\beta l = 0$$

But it is well known that

$$\cos h^2\,\beta l - \sin h^2\,\beta l = 1$$

Hence we get,

$$\cos\beta l\ \cos h\beta l = -1 \qquad (10)$$

This transcendental equation has an infinite number of solutions $\beta i = 1,2,3 \ldots n$.

Corresponding giving an infinite number of natural frequencies,

$$\omega_1 = (\beta_1 l)^2 \sqrt{\frac{EI}{\rho A l^4}} \qquad (11)$$

The first five roots of Eq. (10) are shown in Table I

Table I: Value of Roots

Root	ß1l
1	1.86
2	4.69
3	7.85
4	11.0
5	14.13

The dimensions and the material constant for a cantilever beam studied in this project are shown in Table II

Table II: Experimental Parameter

Parameter	Symbol	Value
Material of beam	MS	-
Total length	L	0.75 m
Width	B	0.015 m
Thickness	T	0.015 m
Moment of inertia	I	4.21875E-09 m4
Young's Modulus	E	2 x 1011N/m2
Mass density	ρ	7830 kg/m3

Putting all required data in Eq. (11) we get the five modes as shown in Table 3.

Table III: Mode Shape Frequency

Mode	Frequency in Hz
1	21.42191
2	136.2003
3	381.5678
4	749.2344
5	1236.28

VI. PROPOSED EXPERIMENTAL SETUP

The proposed experimental setup will be as follows. It consists of a cantilever beam which is connected to FFT analyser through accelerometer.

The frequency of beam will be captured through FFT analyser and the result will be displayed on monitor of a computer.

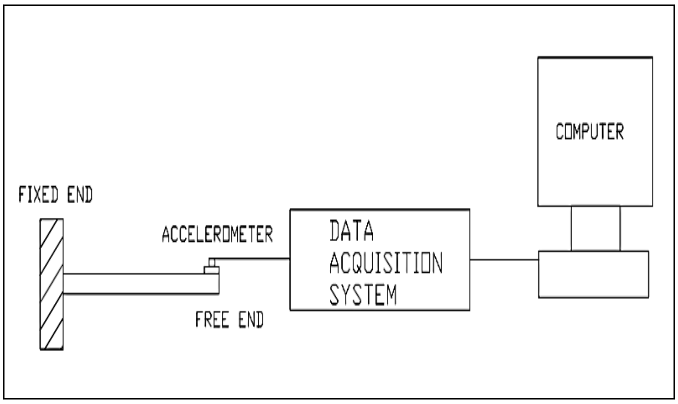

Fig 5 Proposed experimental setup

II. VII CONCLUSIONS

For beams undergoing small displacements, linear beam theory can be used to calculate the natural frequencies, mode shapes, and the response for a given excitation. However, when the displacements are large, linear beam theory fails to accurately describe the dynamic characteristics of the system.

By the literature review it is seen that, compare with previous old systems of vibration analysis of cantilever beam this method identifies the nonlinearities and effects on load deflection characteristics of cantilever. This theoretical study will be main base for further research work. In this report a new design approach for finding the non-linearity in the cracked cantilever beam is analyzed.

VIII FUTURE WORK

- After theoretical study, various parameters of cracked cantilever beam will be found out.
- Theoretical beam design will be done to check structural integrity of beam.
- ANSYS software will be studied to perform the analysis.
- Crack/notch will be developed on cracked cantilever beam.
- Theoretical, numerical and Experimental static analysis of beam will be carried out to find out effect of nonlinearities.
- Experimental verification of numerical results of linear and nonlinear analysis will be performed.

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